

Nucleosynthesis in a simmering universe

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Abstract

Primordial nucleosynthesis is considered a success story of the standard big bang (SBB) cosmology. The cosmological and elementary particle physics parameters are believed to be severely constrained by the requirement of correct abundances of light elements. We explore nucleosynthesis in a class of models very different from SBB. In these models the cosmological scale factor increases linearly with time right through the period during which nucleosynthesis occurs till the present. It turns out that weak interactions remain in thermal equilibrium upto temperatures which are two orders of magnitude lower than the corresponding (weak interaction decoupling) temperature in SBB. Inverse beta decay of the proton can ensure adequate production of several light elements while producing primordial metallicity much higher than that produced in SBB. Other attractive features of these models are the absence of the horizon, flatness and the age problems and consistency with classical cosmological tests.

Early universe nucleosynthesis is regarded as a major “success story” of the standard big bang (SBB) model. The results look rather good and the observed light element abundances are used to severely constrain cosmological and particle physics parameters.

However, there is no object in the universe that has quite the abundance [metallicity] of heavier elements as is produced in the “first three minutes” (or so) in SBB. One relies heavily on success of some kind of re - processing, much later in the history of SBB, to get the low observed metallicity in [eg.] old clusters and inter - stellar clouds. This could [for instance] be in the form of a generation of very short - lived type III stars. Large scale production and recycling of metals through such exploding early generation stars leads to verifiable observational constraints. Such stars would be visible as 27 - 29 magnitude stars appearing any time in every square arc - minute of the sky. Serious doubts have been expressed on the existence and detection of such signals [1].

Of late [2], observations have suggested the need for a careful scrutiny and a possible revision of the status of SBB nucleosynthesis from reported high abundance of 2D in several Ly_α systems. Though the status of these observations is still a matter of debate, and [assuming their confirmation-], attempts to reconcile the cosmological abundance of deuterium and the number of neutrino generations within the framework of SBB are still on, we feel that alternative scenarios should be explored.

Surprisingly, a class of models radically different from the standard one has a promise of producing the correct amount of helium as well as the metallicity observed in low metallicity objects. This paper is a status report on our ongoing efforts to study the cosmological implications of a class of models in which the cosmological scale factor $R(t)$ varies linearly with time. The basic argument is quite straightforward and goes along the lines of STD nucleosynthesis, summarised as follows:

A crucial assumption in the standard model is the existence of thermal equilibrium at temperatures around $10^{12}K$ or $100MeV$. At these temperatures, the universe is assumed to consist of leptons, photons and a contamination of nucleons in thermal equilibrium. The ratio of weak reaction rates of leptons to the rate of expansion of the universe (the Hubble parameter) below $10^{11}K$ (age $\approx .01$ secs) goes as (see eg.[3]):

$$r_w \equiv \frac{\sigma n_l}{H} \approx \left(\frac{T}{10^{10}K}\right)^3 \quad (1)$$

At these temperatures, the small nucleonic contamination begins to shift towards more protons and fewer neutrons because of the n-p mass difference. By $10^{10}K$ i.e. $T_9 \equiv 10$, r_w falls below unity, consequently, the weak interactions fall out of equilibrium and the neutrinos decouple. The distribution function of the ν 's however maintains a Planckian profile as the universe expands. At 5×10^9K (age of about 4 secs), e^+, e^- pairs annihilate. The neutrinos having decoupled, all the entropy of the e^+, e^- before annihilation, goes to heat up the photons - giving the photons some 40% higher temperature than the temperature corresponding to the neutrino Planckian profile. The decoupling of the neutrinos and the annihilation of the e^+, e^- ensures the rapid fall of the neutron production rate $\lambda(p \rightarrow n)$ in comparison to the expansion rate of the universe. n/p ratio freezes to about 1/5 at this epoch. This ratio now falls slowly on account of decay of free neutrons. Meanwhile nuclear reactions and photo - disintegration of light nuclei ensure a dynamic

buffer of light elements with abundances roughly determined by nuclear statistical equilibrium (NSE). Depending on the baryon-entropy ratio, at a critical temperature around $T_9 = 1$, deuterium concentration is large enough for efficient evolution of a whole network of reactions leading up to the formation of the most stable light nucleus, viz. ${}^4\text{He}$. This is the characteristic temperature at which ${}^2\text{D}$ conversion into other nuclei becomes a more efficient channel for the destruction of neutrons than neutron decay. At slightly lower temperatures, deuterium depletion rate becomes small compared to the expansion rate [4] resulting in residual abundances of deuterium and ${}^3\text{He}$. Elaborate numerical codes have been developed [5] to describe the evolution of this phase. The abundances of deuterium, helium - 3, helium - 4 and lithium - 7 can be used to constrain the baryon - entropy ratio, the number of light particles around and the neutrino chemical potential. The primordial metallicity obtained is rather low and one does not see any astrophysical object with metallicity (abundance of lithium - 8 and heavier elements) as low as that predicted by primordial synthesis alone. The oldest objects are believed to be globular clusters. The metallicity reported in these systems is much higher than accounted for by SBB and much too low in comparison with that found in the atmosphere of population I stars and interstellar gas. Special reprocessing and metal enrichment is suggested at a redshift of 10 to 5. No unambiguous experimental signal to this effect has been reported so far [1]. Consistency of the light element abundances in SBB, moreover, is ensured only if the baryonic matter density is some two orders of magnitude less than the closure density. This is regarded as a respite in SBB. Using the rest of the (non-baryonic) matter in a suitable combination of hot and cold dark matter (with possibly a small cosmological constant also thrown in) to build up large scale structures in cosmology has developed into an industry. The current status is not completely satisfactory. In particular, the age estimates of globular clusters are uncomfortably high in comparison with the age of the universe as set by conservative estimates.

Motivated by the above, we explore the possibility of obtaining a consistent scenario for nucleosynthesis in a class of models which are radically different from the standard one. In particular, we consider a cosmological model in which, right through the epoch when $T \approx 10^{12}\text{K}$ and thereafter, the scale factor $R(t)$ increases as t (- the age of the universe). The linear evolution of the scale factor ensures a horizon-free cosmology. We shall later describe models in which such a scaling is possible. With such linear scaling, the present value of the scale parameter, i.e. the present epoch t_o , is exactly determined by the present Hubble constant $H_o = 1/t_o$. The scale factor and the temperature of radiation are related by $RT \approx \text{constant}$ with effect from temperatures $\approx 10^9\text{K}$. This follows from the stress energy conservation and the fact that the baryon - entropy ratio does not change after $kT \approx m_e$ (the rest mass of the electron). From present age and effective CMB temperature (2.7K). one finds the age of the universe when $T \approx 10^{10}\text{K}$ to be of the order of a few years. The universe takes some 10^3 years to cool from 10^{10}K to 10^8K . The rate of expansion of the universe is about 10^7 times slower than the corresponding rates for the same temperature in standard cosmology. This makes a crucial [big] difference and in fact implies that the standard story does not go through.

The process of the neutrinos falling out of thermal equilibrium, for example, is deter-

mined by the rate of ν production per charged lepton:

$$\sigma_{wk}n_l/c^6 \approx g_{wk}\hbar^{-7}(kT)^5/c^6 \quad (2)$$

and the expansion rate of the universe [$H = 1/t$]. Here $g_{wk} \approx 1.4 \times 10^{-45}$ erg- cm^3 . For $kT > m_\mu$, $T > 10^{12}K$

$$\sigma_{wk}n_l/H \approx [\frac{T}{1.62 \times 10^8 K}]^4 \quad (3)$$

Here we have normalised the value of $RT = tT = \text{constant}$ from the value $H_o = 55$ km/sec/Mpc for the Hubble constant - corresponding to $t_o \approx 18 \times 10^9$ years. [$t_9 \approx 2.5 \times 10^9$]. Increasing H_o by a factor of 2 would merely lead to a change in the denominator on the right side of eqn.[3] to $1.8 \times 10^8 K$. When $kT < m_\mu$, the number density of muons is reduced by a factor [$\exp(-m_\mu/kT)$]. Consequently, the rates of weak interactions involving muons get suppressed to

$$\sigma_{wk}n_l/H \approx [\frac{T}{1.62 \times 10^8 K}]^4 \exp[-\frac{10^{12}K}{T}] \quad (4)$$

The corresponding rates in the standard model are:

$$\sigma_{wk}n_l/H \approx [\frac{T}{10^{10}K}]^3 \quad (5)$$

for $kT > m_\mu$, and

$$\sigma_{wk}n_l/H \approx [\frac{T}{10^{10}K}]^3 \exp[-\frac{10^{12}K}{T}] \quad (6)$$

for $kT < m_\mu$. This would lead to the weak interactions maintaining the ν 's in thermal equilibrium to temperatures down to $1.62 \times 10^8 K$. The entropy released from the e^+e^- annihilation heats up all the particles in equilibrium. Both the neutrinos and the photons would therefore get heated up to the same temperature. The temperature then scales by $RT = \text{constant}$ as the universe expands. The relic neutrinos and the photons (the CMBR) would therefore have the same Planckian profile ($T \approx 2.7K$) at present. (The photon number does not significantly change at recombination for a low enough baryon - entropy ratio). This is in marked contrast to the standard result wherein the neutrino temperature is predicted to be lower than the photon temperature. The nuclear reaction rates are simply given by the expressions:

$$\begin{aligned} \lambda(n \longrightarrow p) &= A \int (1 - \frac{m_e^2}{(Q+q)^2})^{1/2} (Q+q)^2 q^2 dq \\ &\times (1 + e^{q/kT})^{-1} (1 + e^{-(Q+q)/kT})^{-1} \end{aligned} \quad (7)$$

$$\lambda(p \longrightarrow n) = A \int (1 - \frac{m_e^2}{(Q+q)^2})^{1/2} (Q+q)^2 q^2 dq$$

These rates have the ratio determined by the neutron - proton mass difference $\equiv Q \approx 15$ [in units $k = T_9 = 1$]:

$$\frac{\lambda(p \longrightarrow n)}{\lambda(n \longrightarrow p)} = \exp(-\frac{Q}{T_9}) \quad (8)$$

The rate of expansion of the universe at a given temperature being much smaller than that in the standard scenario, the nucleons are expected to be in thermal equilibrium with the ratio X_n of neutron number to the total number of all nucleons given by:

$$X_n = \frac{\lambda(p \longrightarrow n)}{\lambda(p \longrightarrow n) + \lambda(n \longrightarrow p)} = [1 + e^{Q/T_9}]^{-1} \quad (9)$$

As in the standard model, Deuterium burning into light elements becomes the more efficient channel for neutron destruction than neutron decay at a temperature $T_9 \approx 1$ and nucleosynthesis commences. [This result follows from a numerical integration of the Boltzmann - rate - equations and was done by using Wagoner's [6] prescription]. At this temperature, one sees from eqn(9) that there are hardly any neutrons left. However weak interactions have not frozen off and inverse beta decay can convert protons into neutrons till temperatures down to $\approx 10^8 K$. The baryonic content of the universe at $T_9 \approx 1$ is constituted by protons (mainly), some neutrons (less than 1%) and a buffer of light elements in NSE. The strength of the buffer is enhanced by fresh neutron formation by the inverse beta decay of the proton and its capture into the buffer by the pn reaction. The buffer depletes by either: (i) the photodisintegration of any light element constituting the buffer followed by the decay of the resulting neutron before it can be recaptured into the buffer by the pn reaction; or (ii) the formation of 4He which is the most stable nucleus at these temperatures. Once helium formation becomes more efficient than neutron decay, most subsequent neutrons formed would precipitate into 4He . This critical epoch of commencement of 4He precipitation is sensitive to the baryon-entropy ratio. If the ratio of number of protons that convert into neutrons after this epoch, to the total baryon number of the universe is roughly 1/8, we would get the observed $\approx 25\%$ 4He . To see this in a little more detail: eqn[8] implies:

$$\frac{\lambda(p \longrightarrow n)}{\lambda(n \longrightarrow p)} = \exp(-\frac{Q}{kT}) \approx e^{-15/T_9} \quad (10)$$

If τ is the neutron life time, eqn(10) gives:

$$\dot{Y}_p \approx -\frac{1}{\tau} e^{-15/T_9} Y_p \quad (11)$$

This is exactly integrated, starting from a temperature T_{9o} , to give:

$$Y_p \approx Y_{po} \exp[-\frac{10^9}{15\tau} e^{-15/T_{9o}}]$$

$Y_{po} - Y_p$ is the number of protons converted to neutrons. If all these protons are converted into neutrons [i.e. T_{9o} is the temperature at the epoch of 4He precipitation as described above], the amount of helium is just:

$$Y_{He} \approx 2[1 - \exp[-\frac{10^9}{15\tau} e^{-15/T_{9o}}]] \quad (12)$$

This is $\approx 24\%$ for $T_{9o} \approx 0.9$. This simply translates into an appropriate requirement on the baryon-entropy ratio. Fortunately one has an extremely user friendly code [5] that we modified to suit the taxing requirements of the much stiffer rate equations that we encounter in our slowly evolving universe. To get convergence of the rate equations for 26 nuclides and a network of 88 reactions [as given in Kawano's code], we executed some 500 iterations at each time step. An additional (89th) reaction (the pp reaction):

$$p + p \longrightarrow D + e^+ + \nu \quad (13)$$

does not decouple on account of the slow expansion of the universe and was incorporated in the code. The results for different values of η are described in table I. We find consistency with the ${}^4\text{He}$ abundances for $\eta \approx 10^{-8}$. The metalicity produced is 8 orders of magnitude greater than the corresponding value one gets in the early universe in the Standard model. This is also a consequence of the slow expansion in this model. A locally higher η in an inhomogeneous model can further enhance metalicity.

To get the observed abundances of light elements besides ${}^4\text{He}$, one would have to fall back upon a host of other mechanisms that were being explored in the SBB in the pre - 1976 days. The most popular processes are: (i) nucleosynthesis by secondary explosions of super massive objects [6], (ii) nucleosynthesis in inhomogeneous models, (iii) effect of inhomogeneous n/p ratios as the universe comes out of the QGP phase transition, (iv) spallation of light nuclei at a much later epoch. It is easy to rule out the survival of ${}^2\text{D}$ by the processes (ii) and (iii) while the process (i) requires very special initial conditions. It also shares a common difficulty with process (iv), viz.: the production of ${}^2\text{D}$ to the required levels is possible but it is accompanied by an overproduction of lithium. Any later destruction of lithium in turn completely destroys ${}^2\text{D}$. Within the framework of the cosmological evolution that we are exploring here, we find the best promise in a model that would combine (ii) and (iv). Table 1 displays the extreme sensitivity of ${}^4\text{He}$ production to η . In an inhomogeneous model with a spatially varying η , there would hardly be any ${}^4\text{He}$ production in a region with η lower by (say) a factor of two. Thus we can have proton rich clouds in low density regions and ${}^4\text{He}$ and metal rich clouds in the higher density regions. The spallation of the former on the later, at a subsequent [cooler] epoch, would produce ${}^2\text{D}$ without the excess production of lithium [7] as lithium forms primarily from spalling ${}^4\text{He}$ over ${}^4\text{He}$.

We feel that one should be able to dynamically account for such conditions within the framework of models we outline in the conclusion.

With $R = t$, the expansion rate does not depend on the background density and thus nucleosynthesis is independent of the number of neutrino species or for that matter to any other (particles) extra degrees of freedom. The age of this universe (defined as the time elapsed from the hot epoch to the present) would be exactly 50% higher than the SBB age determination, $2/3H_o$, from the Hubble parameter.

Conclusion

The purpose of the article is to show that a class of cosmological models can not to be discarded away on account of SBB nucleosynthesis constraints. In any model in which the rate of expansion of the universe is low enough to keep weak interactions in equilibrium at temperatures lower than the ${}^4\text{He}$ precipitating temperature, inverse beta decay can lead

to adequate ${}^4\text{He}$ and metal production. Further, in principal, it is possible to produce ${}^2\text{D}$ by spallation of hydrogen rich clouds over a ${}^4\text{He}$ - metal rich medium at a later epoch.

We finally address the issue of realising the linear evolution within the framework of a Friedman cosmology. Such an evolution can be accounted for in a universe dominated by ‘K - matter’ [8] for which the density scales as R^{-2} . The Hubble diagram (luminosity distance-redshift relation), the angular diameter distance - redshift relation and the galaxy number count-redshift relations do not rule out such a “coasting” cosmology [8,9]. However, if one requires this matter to dominate even during the nucleosynthesis era, the K - matter would almost close the universe. There would hardly be any baryons in the present epoch. An alternative way of achieving a linear evolution of the scale factor is an effective Einstein theory with a repulsive effective gravitational constant at long distances. Such possibilities follow from effective gravitational actions that have been considered in the past [10]. For a fourth order theory with action:

$$S = \int d^4x \sqrt{-g} [\alpha R^2 - \beta R] \quad (14)$$

in the weak field approximation, the effective Newtonian potential is:

$$\phi = -\frac{a}{r} + b \frac{\exp(-\mu r)}{r} \quad (15)$$

For $\mu r \ll 1$ we can have a canonical effective attractive theory. Over large distances, the effective potential is dominated by the first repulsive term alone. A similar possibility occurs in the conformally invariant higher order theory of gravity [11]. Choosing the gravitational action to be the square of the Weyl tensor gives rise to an effective gravity action:

$$S = \int d^4x \sqrt{-g} [\alpha C^2 - \beta R] \quad (16)$$

The dynamics of a conformally flat FRW metric is driven by the anomalous repulsive term alone. Canonical attractive domains occur in the model as non - conformally flat perturbations in the FRW spacetime.

Yet another way of realising a linear evolution of the scale factor is in a class of Brans - Dicke cosmological models [12].

Linear evolution of the scale factor would also be possible in the following “toy” model [13] that combines the Lee - Wick construction of non - topological soliton [NTS] solutions [14] in a variant of an effective gravity model proposed by Zee [15]. Consider the action:

$$S = \int d^4x \sqrt{-g} [U(\phi)R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + L_m] \quad (17)$$

Here ϕ is a scalar field non - minimally coupled to the scalar curvature through the function $U(\phi)$, $V(\phi)$, its effective potential and L_m the matter field action. L_m includes a Higgs coupling of ϕ to a fermion. Let V have a minimum at ϕ_{min} and a zero at ϕ^o . We also choose the Higgs coupling such that the effective fermion mass at $\phi = \phi_{min}$ is greater

than the effective fermion mass at $\phi = \phi^o$. Finally we choose the non - minimal function $U(\phi_{min}) \gg U(\phi^o)$. These conditions are sufficient for the existence of large NTS's with the scalar field trapped at $\phi = \phi^o$ in the interior of a large ball and quickly going to $\phi = \phi_{min}$ across the surface of the ball. With a judicious choice of the surface tension, these balls could be as large as a typical halo of a galaxy. The interior and exterior of such a ball would be regions with effective gravitational constant $[U(\phi^o)]^{-1}$ & $[U(\phi_{min})]^{-1}$ respectively. With $[U(\phi_{min})]$ large enough, the universe would evolve as a curvature dominated universe [without any 'K - matter'].

Such a universe would expand as a Milne universe having canonical gravitating domains restricted to the interior of NTS domains. The interior would have a larger baryon entropy ratio, η , than the exterior. The requirement for the formation and later spallation of 4He deficient clouds onto a 4He rich medium could be realised in such a model.

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TABLE I

Abundances of Some Light Elements and Metals.

η (10^{-9})	^2H (10^{-18})	^3H (10^{-25})	^3He (10^{-14})	^4He (10^{-1})	^7Be (10^{-11})	^8Li & above (10^{-8})
9.0	2.007	1.25	8.65	2.03	1.39	8.06
9.1	2.008	1.26	8.63	2.06	1.32	8.63
9.2	2.009	1.26	8.60	2.10	1.23	9.35
9.3	2.010	1.27	8.59	2.11	1.19	9.75
9.4	2.014	1.26	8.56	2.15	1.11	10.66
9.5	2.015	1.27	8.50	2.18	1.05	11.41
9.6	2.016	1.28	8.52	2.19	1.01	11.88
9.7	2.017	1.28	8.49	2.22	0.96	12.69
9.8	2.020	1.29	8.47	2.25	0.91	13.51
9.9	2.020	1.29	8.45	2.28	0.86	14.47
10.0	2.020	1.30	8.43	2.30	0.83	15.19

Initial Temperature $10^{11} K$ Final Temperature $10^7 K$

No. of iterations at each step 550